nag_airy_ai_deriv (s17ajc)

1. Purpose

nag_airy_ai_deriv (s17ajc) returns a value of the derivative of the Airy function $\mathrm{Ai}(x)$.

2. Specification

```
#include <nag.h>
#include <nags.h>

double nag_airy_ai_deriv(double x, NagError *fail)
```

3. Description

This function evaluates an approximation to the derivative of the Airy function $\mathrm{Ai}(x)$. It is based on a number of Chebyshev expansions.

For large negative arguments, it is impossible to calculate a result for the oscillating function with any accuracy and so the function evaluation will fail. This occurs for $x < (\sqrt{\pi}/\epsilon)^{4/7}$, where ϵ is the ***machine precision***.

For large positive arguments, where Ai' decays in an essentially exponential manner, there is a danger of underflow so the function must fail.

4. Parameters

x

Input: the argument x of the function.

fail

The NAG error parameter, see the Essential Introduction to the NAG C Library.

5. Error Indications and Warnings

NE_REAL_ARG_GT

On entry, **x** must not be greater than ⟨*value*⟩: **x** = ⟨*value*⟩.
x is too large and positive. The function returns zero.

NE_REAL_ARG_LT

On entry, **x** must not be less than ⟨*value*⟩: **x** = ⟨*value*⟩.
x is too large and negative. The function returns zero.

6. Further Comments

6.1. Accuracy

For negative arguments the function is oscillatory and hence absolute error is the appropriate measure. In the positive region the function is essentially exponential in character and here relative error is needed. The absolute error, E, and the relative error, ϵ, are related in principle to the relative error in the argument, δ, by $E \simeq |x^2 \mathrm{Ai}(x)|\,\delta$, $\epsilon \simeq |x^2 \mathrm{Ai}(x)/\mathrm{Ai}'(x)|\,\delta$.

In practice, approximate equality is the best that can be expected. When δ, ϵ or E is of the order of the ***machine precision***, the errors in the result will be somewhat larger.

For small x, positive or negative, errors are strongly attenuated by the function and hence will be roughly bounded by the ***machine precision***.

For moderate to large negative x, the error, like the function, is oscillatory; however, the amplitude of the error grows like $|x|^{7/4}/\sqrt{\pi}$. Therefore it is impossible to calculate the function with any accuracy if $|x|^{7/4} > \sqrt{\pi}/\delta$.

For large positive x, the relative error amplification is considerable, $\epsilon/\delta \simeq \sqrt{x^3}$.

However, very large arguments are not possible due to the danger of underflow. Thus in practice error amplification is limited.

6.2. References

Abramowitz M and Stegun I A (1968) *Handbook of Mathematical Functions* Dover Publications, New York ch 10.4 p 446.

7. See Also

nag_airy_ai (s17agc)

8. Example

The following program reads values of the argument x from a file, evaluates the function at each value of x and prints the results.

8.1. Program Text

```
/* nag_airy_ai_deriv(s17ajc) Example Program
 *
 * Copyright 1990 Numerical Algorithms Group.
 *
 * Mark 2 revised, 1992.
 */

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nags.h>

main()
{
  double x, y;

  /* Skip heading in data file */
  Vscanf("%*[^\n]");
  Vprintf("s17ajc Example Program Results\n");
  Vprintf("     x           y\n");
  while (scanf("%lf", &x) != EOF)
    {
      y = s17ajc(x, NAGERR_DEFAULT);
      Vprintf("%12.3e%12.3e\n", x, y);
    }
  exit(EXIT_SUCCESS);
}
```

8.2. Program Data

```
s17ajc Example Program Data
             -10.0
             -1.0
             0.0
             1.0
             5.0
             10.0
             20.0
```

8.3. Program Results

```
s17ajc Example Program Results
     x           y
  -1.000e+01   9.963e-01
  -1.000e+00  -1.016e-02
   0.000e+00  -2.588e-01
   1.000e+00  -1.591e-01
   5.000e+00  -2.474e-04
   1.000e+01  -3.521e-10
   2.000e+01  -7.586e-27
```